Shortest path for aerial vehicles
*in heterogeneous environment using RRT**

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Context

Onera:

- The first French national aerospace research center;
- Multidisciplinary researches concerning aeronautics and aerospace;
- Trajectory planning for aerial vehicles is an interesting topic. Thus, several approaches have been studied.

PhD thesis:

- This paper is a part of the PhD thesis at Université d'Évry-Val-d'Essonne;
- The objective of the thesis is to find an efficient trajectory planning algorithm for aerial vehicles;
- Constraints related to environment, mission, and obstacles must be considered.

Challenges

Motivations:

- Trajectory planning is a high-demand algorithm for aerial vehicles;
- The existing classical control laws rely on some restrictive approximation. For such complex systems and missions, the optimal problem needs to be considered globally;
- The obstacles induce state constraints that are very difficult to consider with the numerical direct and indirect methods;

Problem statement:

- Non-linear and non-holonomic system:

$$\begin{aligned} x' &= \frac{dx}{ds} = \cos\theta, \\ z' &= \frac{dz}{ds} = \sin\theta, \\ \theta' &= \frac{d\theta}{ds} = c(z)u, \quad |u| \leqslant 1 \end{aligned} \tag{1}$$

  where $u \in \mathbb{R}$ is the control input and $c(z) \in \mathbb{R}_+$ is the maximum curvature that can be performed by the vehicle at the altitude z.

- Heterogeneous environment: decrease of the air density with altitude z, *i.e.* $\rho(z) = \rho_0 e^{-z/z_r}$

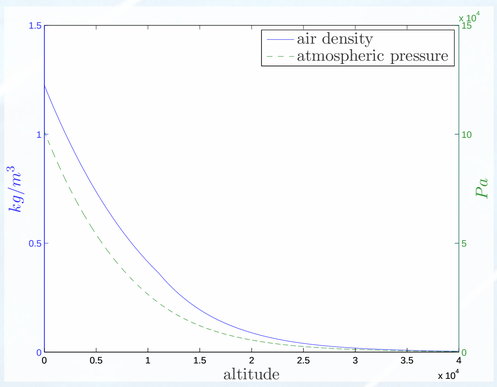


Figure 1: *Air density and atmospheric pressure with respect to altitude (US-76 model)*

$\Rightarrow$ Loss of maneuverability in high altitude:

$$u \prec \rho(z)$$

- Path planning in an environment cluttered with obstacles;
- Optimal trajectory from $\mathbf{x}_{\text{init}}$ to $\mathbb{X}_{\text{goal}}$.

Proposed approach:
The combination of:

- Path planner: The optimal Rapidly-exploring Random Tree (RRT*);
- Optimal control theory: Dubins' paths in heterogeneous environments.

Trajectory planning framework

Optimal Rapidly-exploring Random Trees (RRT*)[1]

An incremental method designed to efficiently explore non-convex high-dimensional spaces by growing the search tree toward large Voronoi areas[2] with the asymptotic optimality property, i.e. almost-sure convergence to an optimal solution.

Principles: Generation of an exploration tree to search the exploration space while looking for the optimal path by verifying, deconnecting and reconnecting branches.

- The tree is expanded towards a randomly generated state $\mathbf{x}_{\text{rand}}$ and obtain $\mathbf{x}_{\text{new}}$;
- The tree grows towards $\mathbf{x}_{\text{new}}$ from the state $\mathbf{x}_{\text{nearest}}$ in the neighborhood $\mathbf{x}_{\text{near}}$ of $\mathbf{x}_{\text{new}}$ whose cost-to-go from $\mathbf{x}_{\text{nearest}}$ to $\mathbf{x}_{\text{new}}$ is the lowest (see Figure 2);
- If there exists a path from $\mathbf{x}_{\text{new}}$ to states in the neighborhood $\mathbf{x}_{\text{near}}$ with less cost-to-go from $\mathbf{x}_{\text{init}}$ to $\mathbf{x}_{\text{new}}$, those paths are replaced (see Figure 3).

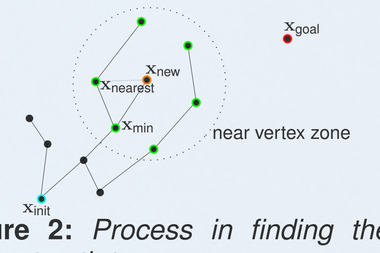


Figure 2: *Process in finding the nearest path to* $\mathbf{x}_{new}$

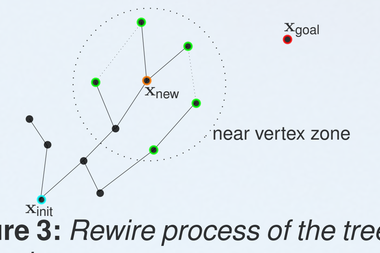


Figure 3: *Rewire process of the tree around* $\mathbf{x}_{new}$

Dubins' paths in heterogeneous environments[3]

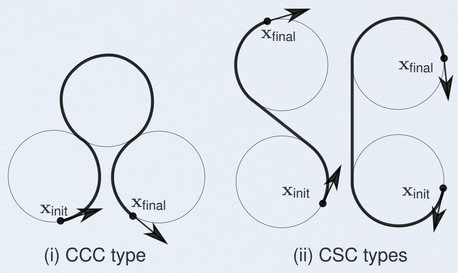


Figure 4: *Dubins' paths*

Dubins' paths illustrated in Figure 4 are the shortest paths between two states of Dubins' vehicle (see equation (1)) based on the optimal control theory[4]. However, they are not realistic for the aerial vehicle traveling in the vertical plane.

Dubins' paths in heterogeneous environments are developed to obtain more realistic path for aerial vehicles. The shortest Dubins' path is used as a metric in the RRT* algorithm as well as the path generation. In the aerial vehicle application, two states are considered far from each other. Thus, only the CSC paths are considered[5] (see Figure 5).

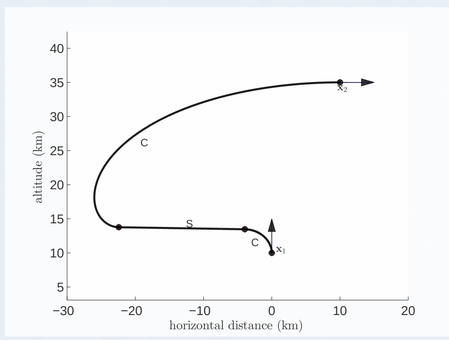


Figure 5: *An example of a Dubins' path of CSC type in heterogeneous environment*

[1] S. Karaman and E. Frazzoli. Optimal kinodynamic motion planning using incremental sampling-based methods, In Proceedings of the International Conference on Decision and Control, Pages 7681-7687, 2010.

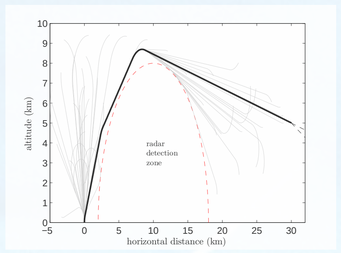
[2] G. Voronoi. Nouvelles applications des paramètres continus à la théorie des formes quadratiques, Journal fur die Reine und Angewandte Mathematik, Vol. 133, Pages 97-178, 1907.

[3] B. Hérissé and R. Pepy. Shortest paths for the dubins' vehicle in heterogeneous environments, In Proceedings of the International Conference on Decision and Control, Pages 4504-4509, 2013.

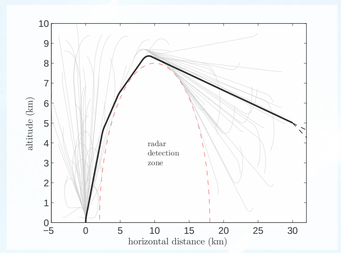
[4] L. E. Dubins. On curves of minimal length with a constraint on average curvature and with presribed initial and terminal position and tangents, American Journal of Mathematics, Vol. 79, Pages 497-516, 1957.

[5] A. M. Shkel and V. Lumelsky. Classification of the dubins set, Robotics and Autonomous Systems, Vol. 34, Pages 179-202, 2001.

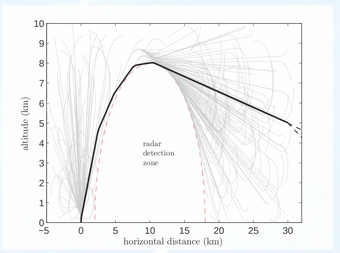
Simulation results



(a) 100 iterations



(b) 150 iterations



(c) 300 iterations

Figure 6: *Exploration tree expansion and results for scenario 1*

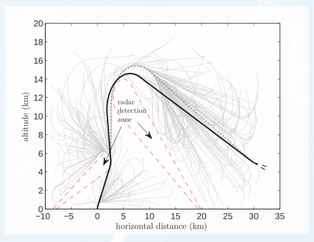


Figure 7: *Simulation result for scenario 2 obtained after 400 iteration*

Perspectives & Future works

- 3D Dubins' paths[6]:

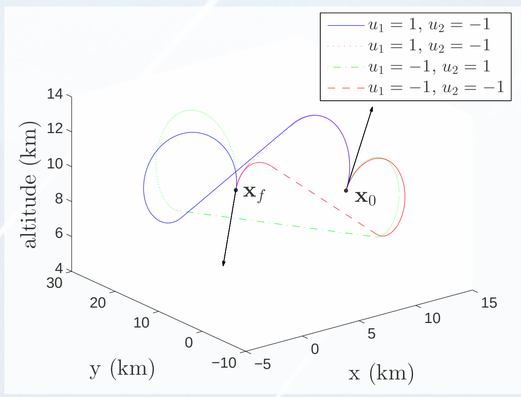


Figure 8: *Four possible CSC paths between two states*

- 3D path planning[7]:

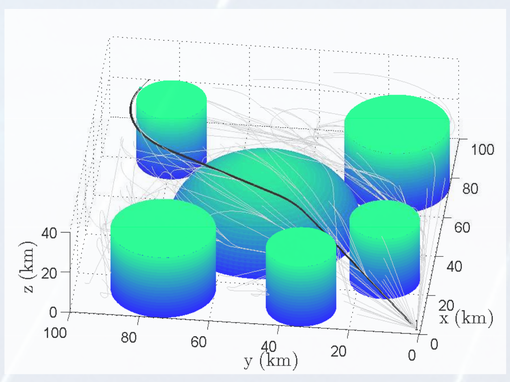


Figure 9: *Exploration trees and results after* 200 *iterations*

- Replanning;
- Real-time constraints for implementation on board the vehicles

[6] P. Pharpatara, B. Hérissé, Y. Bestaoui. 3D-shortest paths for a hypersonic glider in a heterogeneous environment, In Proceedings of the IFAC Workshop on Advances Control and Navigation for Autonomous Aerospace Vehicles , to appear, 2015.

[7] P. Pharpatara, B. Hérissé, Y. Bestaoui. 3D trajectory planning of aerial vehicles using RRT*, IEEE Transaction on Control Systems Technology, submitted.